Econ 802

Second Midterm Exam

Greg Dow November 18, 2020

All questions have equal weight. If you are not making any progress on a question, it is a good idea to move on to something else and come back to it later.

1. Here are some questions about cost curves.

(a) A firm has the short run total cost function $c(y) = A + By + Cy^2$ where $y \geq 0$ is output and $A > 0$, $B > 0$, $C > 0$. Draw a graph showing the following curves: average fixed cost, average variable cost, average total cost, and marginal cost. Be sure the relationships among the curves correctly reflect the nature of this particular cost function, and provide brief explanations.

(b) A firm has a U-shaped long run average cost curve that reaches a minimum at $y_{min} > 0$. Draw a graph of the expansion path for fixed input prices $(w_1, w_2) > 0$. What must be true at the points along the EP where $y < y_{min}$? What about the point on the EP where $y = y_{min}$? What about points along the EP with $y > y_{min}$? Explain.

(c) Professor X says that all firms have horizontal LAC curves, because all firms can scale their inputs and outputs up or down simultaneously in any given proportion by expanding or contracting their current production plan. Discuss this claim.

2. Mr. Clean hates dirt ($x_1 \geq 0$) and trash ($x_2 \geq 0$). His utility function is $u = -x_1^a x_2^b$ where $a > 0$ and $b > 0$.

(a) Draw a graph with $x_1 \geq 0$ on the horizontal axis and $x_2 \geq 0$ on the vertical axis. Show a few indifference curves and indicate the direction of increasing utility. Justify your reasoning about the shapes of the indifference curves.

(b) Mr. Clean currently has the bundle $x_1 = x_2 = 10$. He can hire a cleaning service to remove t_1 units of dirt (where $0 \leq t_1 \leq 10$) and t_2 units of trash (where $0 \leq t_2 \leq 10$). His budget constraint is $p_1t_1 + p_2t_2 \leq m$ where $(p_1, p_2) > 0$ and $m > 0$. Under what conditions will Mr. Clean spend his entire income? Explain carefully.

(c) Suppose Mr. Clean does spend his entire income but he still has $x_1 > 0$ and $x_2 > 0$. Use a graph to explain the nature of the solution to the utility max problem.

3. Gudrun has the indirect utility function $v(p, m) = m / [\sum (p_i/a_i)]$ where $m > 0$ is income, the $a_i > 0$ are constants for $a_1 \ldots a_n$, and the $p_i > 0$ are prices for $p_1 \ldots p_n$.

(a) Calculate the Marshallian demands $x_i(p, m)$ and the Hicksian demands $h_i(p, u)$ for all of the goods $i = 1 \,.\,.\, n$.

(b) Prove that your results from part (a) satisfy the Slutsky equation. It is enough to consider the effect of a change in the price of good i on the quantity of good j.

(c) What is the direct utility function? Note: you will run into problems if you try to minimize $v(p, m)$ with respect to prices. Try guessing the answer and then prove that your guess is correct.

4. Uskali has the utility function $u(c, L) = \ln c + \ln L$ where $c \geq 0$ is consumption and $L \geq 0$ is leisure. He has the time constraint $H + L = T$ where $H \geq 0$ is hours of work and T is total time. He also has the budget constraint $pc = wH + r$ where $p > 0$ is the price of consumption, $w > 0$ is the wage, and $r \geq 0$ is non-labor income.

(a) Solve for the demand functions $c(p, w, r)$ and $L(p, w, r)$. Explain your reasoning.

(b) In this part, you can ignore the previous utility function. Assume that a consumer cares about many consumption goods $(c_1 \,.\,.\, c_k)$ as well as leisure. Describe a set of assumptions that could be used to combine the individual consumption goods into a single good C and then write utility as a function of the composite good C and leisure. Use some mathematics to show how you would do this.

(c) Suppose there is a single consumption good but there are many consumers $i = 1 \,.\,.\, n$ with identical utility functions $u_i = \ln c_i + \ln L_i$. If your calculations in part (a) are correct, you can compute aggregate demands for consumption and leisure of the form $C(p, w, R)$ and $L(p, w, R)$ where $R = \sum r_i$ is aggregate non-labor income. But if you compute the individual indirect utility functions you will find that they are not in the Gorman form. How can this apparent contradiction be resolved?

5. A smart undergraduate economics student makes the following statements. Give a clear response in each case. Use any combination of math, graphs, and words.

(a) Everyone knows that demand curves can slope down. The possibility of a Giffen good shows that demand curves can also slope up. Thus, the theory of consumer behavior does not make any testable predictions.

(b) Economists sometimes propose raising the price of gas so that people will drive less. But they also sometimes propose compensating consumers by giving them enough additional income that they can afford their original consumption bundle. Overall, these two things cancel out and consumer behavior will not change.

(c) Firms minimize cost for a given output level. Consumers minimize expenditure for a given utility level. Therefore the theories of the firm and the consumer are identical.